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Theory and Methodology

The choice of a technical efficiency measure on the free disposal hull reference technology: A comparison using US banking data

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Abstract

This paper evaluates a variety of technical efficiency measures based on a given nonparametric reference technology, the free disposal hull (FDH). Specifically, we consider the radial measure of Debreu (1951)/Farrell (1957) and the nonradial measures of Färe (1975), Färe and Lovell (1978) and Zieschang (1984). Furthermore, input-based, output-based, and graph efficiency versions of these four measures are computed. Theoretical consideration as to the best choice among these alternative measures is inconclusive; therefore, we examine this problem from an empirical viewpoint. Calculating thirteen different measures of technical efficiency for a sample of US banks, we compare the measures' efficiency distributions and rankings, paying particular attention to how well the radial measure approximates its nonradial alternatives. © 1998 Elsevier Science B.V.

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1. Introduction

Technical efficiency refers to the ability of an organization to operate on the boundary of its production possibilities set. In recent years a substantial body of literature on the theoretical and empirical measurement of technical efficiency has been generated by researchers in a wide range of fields. [1] Two critical issues are associated with measuring efficiency – appropriate specification of the underlying technology relative to which efficiency is assessed and suitable quantification of the distance between an observation and the reference technology. The latter issue itself usually involves two choices: choice of measure and choice of orientation. Not surprisingly, these issues have received considerable attention in the literature. However, while the choice of the reference technology has been examined thoroughly from both theoretical and empirical perspectives, [2] the issue of how best to measure distance from the frontier has been confined largely to the theoretical literature.

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[1] See Lovell (1993) and Seiford (1996) for discussions and extensive bibliographies of this literature.

[2] E.g., Grosskopf (1986) shows that the choice among deterministic nonparametric reference technologies systematically affects the magnitudes of technical efficiency calculated.

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This article is concerned with the second aspect of efficiency measurement – how to measure an observation's distance from the reference technology. In empirical work, the (often input-based) radial measures of efficiency have become the standard. However, the theoretical literature offers a plethora of alternative nonradial efficiency indices. This article focuses on the nonradial measures proposed by Färe (1975), Färe and Lovell (1978), Färe et al. (1983), Russell (1985), and Zieschang (1984); a number of newer measures, surveyed by Pastor (1995), are not discussed. The primary motive in proposing the nonradial alternatives is a fundamental conflict between the radial measures of technical efficiency proposed by Debreu (1951) and Farrell (1957), and the intuitive Koopmans (1951) definition of technical efficiency. Debreu–Farrell measures implicitly define technical efficiency relative to the isoquant, whereas the Koopmans definition equates technical efficiency with membership of the efficient subset of technology. Despite the theoretical developments, most empirical work ignores the nonradial alternatives. [3]

This article empirically implements and evaluates a wide variety of radial and nonradial efficiency measures relative to a given reference technology, the nonparametric-deterministic free disposal hull (FDH) of Deprins et al. (1984). [4] The conflict between the radial technical efficiency measures and the Koopmans (1951) definition of technical efficiency is especially pronounced for the FDH technology, making it a good case to study. The first goal of this paper is to use the FDH reference technology to compare the performances of four alternative measures of technical efficiency – the radial measure of Debreu (1951)/Farrell (1957), and the nonradial measures introduced by Färe (1975), Färe and Lovell (1978) and Zieschang (1984).

The second, closely related, goal is to investigate the effect of the orientation of a technical efficiency measure on resulting efficiency scores. In particular, in addition to the traditional input-based and output-based orientations, we also consider the graph versions of each of the four efficiency measures mentioned above. Input-based measures proportionally shrink an observation's input vector to the point where the observed output vector is still just feasible; these measures are 'oriented' in the input-dimension only. Output-based measures expand an output vector radially until it just remains feasible; these measures are 'oriented' in the output-dimension only. By contrast, graph measures allow simultaneous decreases in the inputs and increases in the outputs when projecting an observation to the efficient frontier. The rationale for including graph efficiency measures in our analysis is to meet Koopmans' definition of efficiency as closely as possible. This is also the primary rationale underlying many of the more recently proposed efficiency measures; e.g., Ali and Lerme (1991), Bardhan et al. (1994), Lovell and Pastor (1994), Lovell et al. (1995), and Tone (1993). These measures have been characterized as 'global' efficiency measures because they treat all input- and output-dimensions simultaneously. For a number of reasons, the most recently proposed measures are not treated in either the theoretical or empirical sections of this article. First, many of the newest measures do not have 'oriented' counterparts; this would make it impossible to explore the effect of orientation choice. Second, many of these measures have been reviewed elsewhere; e.g., Pastor (1995). Finally, including all of the various efficiency measures would render the empirical portion of the analysis unwieldy.

The choice of orientation has practical, as well as, theoretical implications. In some applications choice of orientation is clear. For example, in Indian sugar processing, plants have very little control over their choice of capital, labor, or sugar cane inputs (due to institutional arrangements); meanwhile India has recently become a net importer of sugar, spurring interest in expanding the domestic production of sugar (see Ferrantino and Ferrier, 1995). In this case, an output orientation was the logical choice. In the field of health care, on the other hand, where the emphasis is on cost-control, the 'natural' choice would be an input-orientation (see Ferrier and Valdmanis, 1996). However, some recent research has voiced concern that restricting attention to a particu-

[3] There are some notable exceptions. For example, Deller and Nelson (1991) used the Färe and Lovell (1978) measure, while Lovell and Pastor (1994) and Lovell et al. (1995) use the 'global efficiency measure', or GEM.

[4] A similar analysis for data envelopment analysis (DEA) models is reported in Ferrier et al. (1994).

lar orientation (e.g., input-based efficiency measurement) may neglect major sources of technical inefficiency in the other direction (e.g., outputs) (see Berger et al., 1993 who raise this issue for the US banking industry). After reviewing the theoretical literature, which is inconclusive as to the best choice among the alternative efficiency measures and orientations of measurement, we examine the problem from an empirical viewpoint. By investigating whether these efficiency measures yield different empirical distributions and rankings, and examining how well the radial efficiency measure approximates the nonradial alternatives, this research sheds light on the issue of how the choice of measure affects efficiency evaluation.

The remainder of this article proceeds as follows. Section 2 reviews the theoretical debate on the measurement of technical efficiency and defines the efficiency measures considered in the empirical analysis (Section 4). Section 3 discusses the FDH reference technology and the calculation of the various efficiency indices relative to it. Section 4 calculates the technical efficiency of a sample of US banks using the four different measures, each under all three different orientations, and compares the resulting efficiency scores. Further reflections and conclusions are provided in Section 5. To the best of our knowledge this is the most extensive systematic empirical comparison of such a broad set of radial and nonradial efficiency measures under different orientations.

2. The free disposal hull reference technology

The nonparametric approach to efficiency measurement typically makes very weak assumptions on the underlying reference technology relative to which efficiency is measured. [5] Among the various possible reference technologies, FDH imposes perhaps the mildest assumptions. Specifically, aside from the usual regularity axioms (i.e., 'no free lunch', the possibility of inactivity, boundedness, and closedness), FDH imposes only strong free disposability in inputs (i.e., positive monotonicity) and in outputs (i.e., nestedness of input requirement sets). The latter two conditions imply that an increase in inputs cannot result in a decrease in output and that any reduction in outputs remains producible given the same set of inputs. Note that these conditions allow for variable returns to scale in production.

A production technology transforms the nonnegative inputs $x=(x_1,x_2,\dots,x_m)\in\mathbb{R}^m_+$ into the nonnegative outputs $y=(y_1,y_2,\dots,y_n)\in\mathbb{R}^n_+$. As mentioned above, the choice of orientation is sometimes evident from the problem at hand. For example, if output levels are exogenous to the decision making units while the inputs are under their control, then an input-orientation is appropriate. However, at times the choice of orientation is more difficult, as it may be unclear which variables are discretionary. To allow for different measurement orientations, technology is defined from three perspectives – input space, output space, and graph space.

For the input-based measures of technical efficiency, technology can be represented by the input correspondence, $y\to \mathrm{L}(y)\subseteq\mathbb{R}^m_+$, which assigns an output vector y to the subset of all input vectors x that can produce it. The input correspondence of the FDH reference technology defines a piecewise linear technology constructed on the basis of observed input-output combinations:

$$\mathrm{L}(y)^{\mathrm{FDH}}=\{x\mid x\in\mathbb{R}^m_+,\ zN\geq y,\ zM\leq x,\ zI_k=1,\ z_i\in\{0,1\}\}.$$

The $k\times n$ matrix N contains the n observed outputs of each on the k observations in the data set, M is the $k\times m$ matrix of observed inputs, z is a $1\times k$ vector of intensity parameters, and I_k is a $k\times 1$ vector of ones. Similarly, the output correspondence maps inputs x into subsets $\mathrm{P}(x)\subseteq\mathbb{R}^n_+$ of outputs; in the case of the FDH technology it is defined as:

$$\mathrm{P}(x)^{\mathrm{FDH}}=\{y\mid y\in\mathbb{R}^m_+,\ zN\geq y,\ zM\leq x,\ zI_k=1,\ z_i\in\{0,1\}\}.$$

Finally, technology can be represented by its graph or transformation set; i.e., the set of all feasible input-output vectors. The graph of the FDH reference technology is given by:

$$\mathrm{GR}^{\mathrm{FDH}}=\{(x,y)\mid x\in\mathbb{R}^m_+,\ y\in\mathbb{R}^n_+,\ zN\geq y,\ zM\leq x,\ zI_k=1,\ z_i\in\{0,1\}\},$$

[5] These assumptions are generally less restrictive than those used in parametric approaches (see Lovell, 1993 for details).

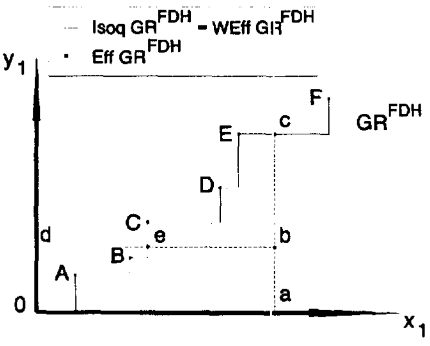

Fig. 1. FDH graph section.

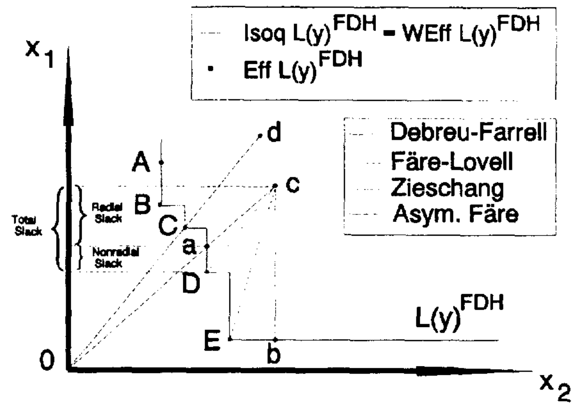

Fig. 2. FDH input section.

and serves as the reference technology for the graph measures of technical efficiency.

Consistent with variable returns to scale, the elements of the intensity vector z are restricted to sum to unity. Because the intensity vector contains only zeros or ones, linear combinations of multiple observations are excluded and convexity is not imposed on the technology. [6] This restriction is the crucial (and only) difference between FDH and the widely used variable returns to scale data envelopment analysis (DEA) technology with strong input and output disposability (Banker et al., 1984). To develop an intuition for the FDH reference technology, note that each activity spans one orthant, positive in the inputs and negative in the outputs, reflecting free disposal in inputs and outputs. The FDH reference technology is the boundary of the union of all such orthants. Its graph and isoquants typically follow stair-step patterns. A typical graph section and an isoquant are shown in Figs. 1 and 2, respectively.

Though not as popular as DEA in applied work, FDH provides an attractive basis for the evaluation of the different efficiency measures for three reasons. First, it imposes minimal assumptions with respect to the production technology. Second, because the conflict between the radial measure of technical efficiency and Koopmans definition of technical efficiency can be quite prominent for the FDH reference technology, it provides a good test case for examining empirical differences across radial and nonradial measures of efficiency. Finally, on FDH the conflict between the traditional input- and output-based and Koopmans notions of efficiency highlight the need to reconsider the overwhelming popularity of the input- and output-oriented measures of technical efficiency. The Koopmans definition, in fact, would give priority to graph efficiency measures. Thus, a comparison between input-based, output-based and graph measures of technical efficiency on FDH seems warranted. The second and third reasons will become more evident in the next section.

While FDH is very intuitive and attractive for efficiency measurement purposes, it does possess some drawbacks. [7] First, strong disposability assumptions preclude the detection of congestion on the technology. In contrast, some DEA models can accommodate for this phenomenon. [8] Furthermore, the integer condition on solutions under FDH results in a loss of contact with the duality theory of ordi-

[6] While the intensity vector contains only the integers 0 and 1, the mixed integer programming problems for computing efficiency scores under FDH can be easily solved using a data classification algorithm based on simple vector dominance reasoning (see Tulkens, 1993). A detailed description of the algorithms used to compute the efficiency measures is provided in an appendix, which is available upon request.

[7] The theoretical and empirical advantages and disadvantages of FDH relative to the DEA family of nonparametric reference technologies are extensively discussed by Lovell and Vanden Eeckaut (1994) and Tulkens (1993).

[8] The known technologies that allow for congestion combine the assumptions of ray-monotonicity and convexity. Thus, a formulation of congestion for FDH, which does not impose convexity, is lacking.

nary linear programming. As a consequence, FDH offers little information regarding the underlying structure of production technology (e.g., opportunity costs, substitution ratios, etc.). This is in contrast with DEA models which allow one to determine substitution and transformation possibilities through duality theory.

3. Alternative measures of technical efficiency

Two different notions of technical efficiency have emerged in the economics literature. The first, due to Debreu (1951) and Farrell (1957), is based on radial measures of technical efficiency. In the input-based case, Debreu–Farrell define technical efficiency as one minus the maximum equiproportionate reduction in all inputs that still allows production of the given outputs. The second notion, introduced by Koopmans (1951), defines a producer as technically efficient if an increase in any output requires a decrease in at least one other output, or if a decrease in any input requires an increase in at least one other input. The great intuitive appeal of this definition has led to its adoption by several authors, including Charnes et al. (1978) and Färe and Lovell (1978).

Input-based, radial efficiency measures shrink the input vector, holding input-mix and the output vector constant, until it is still just feasible to produce the observed vector of outputs. Analogous output-based, radial measures also exist. These two measures are each oriented in a single space-input space or output space, respectively. Graph measures of technical efficiency allow for the simultaneous adjustment of both inputs and outputs. For ease of exposition, the following discussion initially concentrates on input-based efficiency measures; output-based and graph efficiency measures are considered near the end of this section.

3.1. Subsets of technology

To better understand the distinction between the two notions of technical efficiency, we formalize how the subsets of the reference technologies are defined. Different measures of technical efficiency relate observations to different subsets of the input correspondence. Three subsets of $L(y)$ merit particular attention (see Färe et al., 1994). First, the input isoquant of the input correspondence:

$$\operatorname{Isoq} L(y) = \{x \mid x \in L(y),\ \lambda x \notin L(y) \text{ for } \lambda \in [0,1)\};$$

second, the weak efficient subset of the input correspondence: [9]

$$\operatorname{WEff} L(y) = \{x \mid x \in L(y),\ x' <^{*} x \Rightarrow x' \notin L(y)\};$$

and, finally, the efficient subset of the input correspondence:

$$\operatorname{Eff} L(y) = \{x \mid x \in L(y),\ x' \leq x \Rightarrow x' \notin L(y)\}.$$

These subsets are related as follows: $\operatorname{Isoq} L(y) \supseteq \operatorname{WEff} L(y) \supseteq \operatorname{Eff} L(y)$.

The Koopmans notion of efficiency is much more demanding than the Debreu–Farrell efficiency measure. While the Koopmans definition requires productive activities to be elements of the efficient subset, the Debreu–Farrell measure requires only that efficient observations belong to the isoquant, though not necessarily to the efficient subset. Consequently, any reference technology for which the isoquant diverges from the efficient subset highlights the conflict between these two concepts of technical efficiency. For many of the popular reference technologies used in the programming approach (e.g., the DEA models), the isoquant and the efficient subset diverge (see Färe et al., 1994), therefore this problem deserves serious attention. Under FDH the incongruity between the two notions of technical efficiency is particularly relevant. Due to the strong disposability of inputs, the isoquant and the weak efficient subset coincide under the FDH reference technology; however (as is evident from Figs. 1 and 2), the efficient subset only contains disjoint points. In Fig. 2, the input efficient subset is simply the set of productive activities {B, C, D, E}. The distinction between the isoquant and the efficient subset is thus very pronounced (especially when compared to the DEA reference technologies).

[9] The vector inequality conventions used in the text are as follows: $x \geq y$ if and only if $x_i \geq y_i$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ for all i; and $x >^{*} y$ if and only if $x_i > y_i$ or $x_i = y_i = 0$ for all i.

In fact, Koopmans (1951, p. 60 and 80) required simultaneous membership in the efficient subsets of both the input and the output correspondences (or, synonymously, the graph efficient subset). Much of the theoretical literature on technical efficiency measures, however, has focused on membership in either one of these two efficient subsets. Many of the oriented efficiency measures proposed to remedy defects of the Debreu–Farrell efficiency measure succeed in measuring efficiency relative to an efficient subset, but only the one subset corresponding to its orientation. Thus, at best the oriented alternatives only partially meet the Koopmans requirement for efficiency. However, some of the graph measures of efficiency satisfy the Koopmans requirement; this bestows an advantage on these graph measures that the input- and output-oriented measures do not possess.

For only a few nonparametric reference technologies (e.g., the constant returns to scale DEA model with strong disposability in inputs and outputs of Charnes et al., 1978 and the constant returns to scale version of the homothetic nonparametric models of Primont and Primont, 1994) does membership of either the input or the output efficient subset imply that an observation is in the graph efficient subset. [10] For most nonparametric reference technologies, FDH in particular, the divergence between the efficient subsets of the input and output correspondences is potentially very important. Fig. 1 shows that radial measures in either the input- or output-orientation are unlikely to project an inefficient observation onto the graph efficient subset of FDH. This reenforces the case in favor of graph efficiency measures, which guarantee membership in the graph efficient subset of the FDH, so as to meet the Koopmans definition of technical efficiency as closely as possible.

3.2. Desirable properties of efficiency measures

Addressing the conflict between radial measures of efficiency and the Koopmans definition of efficiency, Färe and Lovell's (1978) initiated a literature on the axiomatic approach to technical efficiency measurement. They proposed a set of desirable properties that a measure of technical efficiency measure should possess. In terms of an input-based measure of technical efficiency, $E_i(x,y)$, Färe and Lovell's (1978) list of four desirable properties is as follows:

(P1) Input vectors should be judged efficient if and only if they belong to the efficient subset:

$$\text{If } x \in L(y),\ y > 0, \text{ then } E_i(x,y) = 1 \Leftrightarrow x \in \text{Eff}\,L(y);$$

(P2) Inefficient input vectors should be compared to vectors belonging to the efficient subset:

$$\text{If } x \in L(y),\ y > 0, \text{ and } x \notin \text{Eff}\,L(y), \text{ then } E_i(x,y) \text{ should compare } x \text{ to some } x^* \in \text{Eff}\,L(y);$$

(P3) $E_i(x,y)$ should be homogeneous of degree minus one (i.e., a feasible scaling of the input vector leads to an inverse scaling of the efficiency measure):

$$\text{If } x \in L(y), \text{ and } \lambda x \in L(y),\ y > 0, \text{ then } E_i(\lambda x,y) = \lambda^{-1} E_i(x,y) \text{ for all } \lambda \in [\lambda^\circ, +\infty], \text{ where } \lambda^\circ x \in \text{Isoq}\,L(y);$$

(P4) $E_i(x,y)$ should satisfy strict negative monotonicity (i.e., increasing one input while holding all other inputs and all outputs constant cannot increase the efficiency measure):

$$\text{If } x \in L(y),\ y > 0, \text{ and } x' \geq x, \text{ then } E_i(x,y) \geq E_i(x',y).$$

A weaker version of property (P4) is given by:

(P4′) $E_i(x,y)$ should satisfy weak negative monotonicity (i.e., increasing one input while holding all other inputs and all outputs constant lowers the efficiency measure):

$$\text{If } x \in L(y),\ y > 0, \text{ and } x' \geq x, \text{ then } E_i(x,y) \geqslant E_i(x',y).$$

A fifth desirable property that has appeared in the literature is that of commensurability or units invariance. This property requires the measure of efficiency to be independent of the units in which the inputs and outputs are measured (see Färe et al., 1994):

[10] Formally, $x \in \text{Eff}\,L(y) \Leftrightarrow y \in \text{Eff}\,P(x) \Leftrightarrow (x,y) \in \text{Eff}\,GR$ requires the existence of a joint efficiency production function, which imposes strict monotonicity on the production correspondences (see Färe, 1983 for details).

(P5) $E_i(x, y)$ should be units invariant:

If $x \in L(y)$ and $x' \in L(y)$ for $x' = Ax$, $y' = By$,

then $E_i(x, y) = E_i(x', y')$,

where A and B are positive diagonal matrices. [11]

Properties (P1) and (P2) assure at least partial compliance with the Koopmans definition of efficiency. Properties (P3), (P4) and (P4′) address the sensitivity of the efficiency measure with respect to input usage. The third property imposes a direct proportionality between the level of all inputs used and technical efficiency; the fourth insures that technical efficiency is sensitive to the level of any single input used. [12] Property (P5) assures that efficiency measure can not be 'gamed' by changing the units of measure for the inputs and/or outputs change. The additive model of Charnes et al. (1985) satisfies the Koopmans definition of efficiency, but does not satisfy (P5). Lovell and Pastor (1995), however, show that any efficiency measure can satisfy (P5) provided that the appropriate model is considered (in DEA as well as in FDH); all four of the efficiency measures considered in this article are units invariant. [13]

3.3. Radial and nonradial, input-based measures of technical efficiency [14,15]

The input-based radial measure of technical efficiency introduced by Debreu (1951) and Farrell (1957) is given by:

$$DF_i(x, y) = \min\{\lambda \mid \lambda \geq 0,\ \lambda x \in L(y)\}.$$

As is true of all of the measures of technical efficiency discussed in this section, $DF_i(x, y)$ varies between zero and one. A value of unity is a necessary, though not sufficient, condition for Koopmans efficiency; a value of unity and the absence of (nonradial) slack is a sufficient condition for Koopmans efficiency. $DF_i(x, y)$ indicates the proportion of the observed inputs necessary to produce the observed level of outputs. Note that $DF_i(x, y)$ assumes the isoquant as the relevant subset of technology for defining technical efficiency. An observation is judged efficient by the radial input efficiency measure if and only if it belongs to the isoquant; it is inefficient otherwise. Assuming constant input prices, $(1 - 1/DF_i(x, y))$ gives the proportion by which observed cost exceeds minimal cost. This straightforward cost interpretation is one of the advantages of the radial measures of technical efficiency.

The nonradial, input-based Färe and Lovell (1978) measure of technical efficiency generalizes $DF_i(x, y)$

[11] The units invariance property is easily illustrated by referring to the linear programs (LPs) used to compute the efficiency measures on FDH (see the appendix); changing units of measurements amounts to making elementary transformations on the constraints in the LPs and does not have an impact on solutions to the LPs.

[12] The efficiency measures based directly on the objection function of the additive model, such as the one presented in Lovell et al. (1995), are strictly monotonic (see Pastor, 1995).

[13] A sixth desirable property, translation invariance, has been introduced in the literature (Ali and Seiford, 1990; Pastor, 1997). This is of theoretical interest since few existing efficiency measures are invariant to an affine translation of the input and output data; in fact, none of the four efficiency measures considered in this article satisfy this property. However, the translation invariance property is of concern only if the data are not all strictly positive (see Lovell et al., 1995). Since the data used in the empirical analysis in Section 3 do not contain any negative values (zero values do occur; see footnote 14 for their treatment), this issue of less importance in the present case. For the use of negative data with DEA, see Pastor (1994).

[14] In presenting the efficiency measures we assume strictly positive input and output vectors to reduce notational clutter. For semipositive input and output vectors the definitions must be modified so as to eliminate the impact of zeros (see Färe et al., 1983); the empirical application in Section 4 incorporates these modifications.

[15] Note that a multiplicity of technical efficiency measures is possible due to three interrelated factors (see Färe et al., 1983). First, there are three subsets of the input correspondence against which the technical efficiency of an activity can be gauged. Second, these subsets are unlikely to be singletons, which in general leaves a choice among its elements. Third, the size of each of these subsets depends on the assumptions made on the structure of the production technology. The general problem is therefore how to define 'the' measure of technical efficiency that relates an inefficient observation to an element of a subset of the input correspondence in an economically meaningful way.

by allowing a different scaling of each individual input. [16] This measure reduces all nonzero inputs by the set of individual factors that minimize the arithmetic mean of the reductions. By doing so, it ensures that the resulting input vector is an element of efficient subset of the input correspondence, Eff L(y), though nonradial output slacks may be present. The Färe and Lovell (1978) measure is given by:

$$\mathrm{FL}_i(x,y) = \min\left\{ \sum_{i=1}^{m} \lambda_i/m \mid (\lambda_1 x_1,\ldots,\lambda_m x_m) \in \mathrm{L}(y),\ \lambda_i \in (0,1] \right\}.$$

The Zieschang (1984) nonradial, input-oriented measure of technical efficiency is:

$$\mathrm{Z}_i(x,y) = \mathrm{FL}_i(x\cdot \mathrm{DF}_i^+[x,y],y)\cdot DF_i^+(x,y)$$

where

$$\mathrm{DF}_i^+(x,y) = \min\{\lambda \mid \lambda \geq 0,\ \lambda x \in \mathrm{L}^+(y) = \mathrm{L}(y) + \mathbb{R}_+^m\}.$$

$\mathrm{Z}_i(x,y)$ is an amalgam of the Debreu–Farrell and Färe–Lovell measures. [17] It first radially scales the inefficient observation down to the isoquant, and then shrinks the resulting input vector until an element in the efficient subset is reached. The first step holds input mix constant; the second step removes any remaining inefficiency due to 'slack' (i.e., inefficiency in the mix of inputs). In this regard, $\mathrm{Z}_i(x,y)$ is similar to the two stage optimization of the standard DEA model, though it differs from DEA in that it does not account for output slacks. Because it may leave slack in the output dimensions, $\mathrm{Z}_i(x,y)$ does not guarantee full compliance with the Koopmans definition of efficiency.

Finally, the nonradial, input-based asymmetric Färe measure (Färe, 1975; Färe et al., 1983) of technical efficiency is defined as:

$$\mathrm{AF}_i(x,y) = \min\{\mathrm{AF}_i^j(x,y)\} \quad j=1,\ldots,m,$$

where

$$\mathrm{AF}_i^1(x,y) = \min\{\lambda_1 \mid (\lambda_1 x_1,\ldots,x_j,\ldots,x_m) \in \mathrm{L}(y)\}$$

$$\vdots$$

$$\mathrm{AF}_i^m(x,y) = \min\{\lambda_m \mid (x_1,\ldots,x_j,\ldots,\lambda_m x_m) \in \mathrm{L}(y)\}.$$

$\mathrm{AF}_i(x,y)$ scales down each input in turn, holding outputs and the other inputs fixed, and then takes the minimum over all m of these scalings. Thus, $\mathrm{AF}_i(x,y)$ seeks the shortest, uni-dimensional distance to the frontier. [18] Note that this measure scales inefficient observations down to the boundary of L(y), which need not coincide with any of its subsets.

A potentially important distinction between radial and nonradial efficiency measures is that the latter allow for technical inefficiencies resulting from wrong choices of the input mix. By contrast, the radial efficiency measure evaluates efficiency along a ray. Therefore, it holds factor proportions fixed and, at least implicitly, ignores, or assumes the absence of, any inefficiency in the input mix. The inefficiency in input mix appears as 'slack' in the radially adjusted input vector. Of course, given price data the inefficiencies in input mix can be quantified by a separate measure of allocative, as opposed to technical, efficiency (see Färe et al., 1985, 1994).

A number of relationships among the efficiency measures are worth noting. First, in the special case of a single input, all of the measures coincide. Second, with multiple inputs, $\mathrm{DF}_i(x,y)$ is still a special case of $\mathrm{FL}_i(x,y)$ as is $\mathrm{AF}_i(x,y)$. $\mathrm{DF}_i(x,y)$ is the special case of $\mathrm{FL}_i(x,y)$ in which $\lambda_1 = \lambda_2 = \cdots = \lambda_m$; $\mathrm{AF}_i(x,y)$ is the special case in which $\lambda_j = 1$ for all j such that $\mathrm{AF}_i^j \neq \min\{\mathrm{AF}_i^k\}$, $k = 1,\ldots,m$. Furthermore, only $\mathrm{Z}_i(x,y)$ is defined with

[16] $\mathrm{FL}_i(x,y)$ is also known as the Russell efficiency measure (see Färe et al., 1985). A similar measure has been proposed in Bardhan et al. (1994). Bardhan et al. (1994) also discusses output-oriented and graph versions of the same measure (see Section 3.4 below).

[17] The Debreu–Farrell component is calculated on a technology satisfying strong input disposability. Note that radial measures have been defined for both weakly and strongly disposable technologies (see Färe et al., 1994).

[18] Thanassoulis and Dyson (1992) propose a modification of DEA that is a special case of the asymmetric Färe measure. Rather than scaling all inputs back to the frontier and then selecting the smallest such scaling, they allow decision makers to give 'pre-emptive priority' to a particular input or output dimension based on their preferences and the efficiency score is then calculated with respect to that one dimension only.

the specific intention of eliminating slacks. [19] Consequently, if $DF_i(x,y)$ scales an inefficient observation down to the efficient subset, then it coincides with $Z_i(x,y)$ (i.e., the $FL_i(x,y)$ component of $Z_i(x,y)$ equals unity). Thus, a comparison of $DF_i(x,y)$ and $Z_i(x,y)$ is an easy way to detect the presence of slack. Finally, for a given reference technology, a complete ordering among these efficiency measures is possible (Färe et al., 1983; and Kerstens and Vanden Eeckaut, 1995): $DF_i(x,y) \geq Z_i(x,y) \geq FL_i(x,y) \geq AF_i(x,y)$.

Fig. 2 illustrates these four efficiency measures. The radial measure, $DF_i(x,y)$, scales inefficient observations down to the isoquant (e.g., see observation c). Thus, only those observations that lie on a ray through one of the elements of the efficient subset (e.g., observation d) are scaled down to the efficient subset. The probability of this occurring in empirical applications is likely to be low. $FL_i(x,y)$ scales the inefficient observation c down to observation E. $Z_i(x,y)$ relates the inefficient observation c to observation D by adjusting the radial efficiency measure for the remaining slack in the first input. Finally, $AF_i(x,y)$ selects b as a reference point for observation c, since point c's performance is worst in the first input dimension. Note that $AF_i(x,y)$ leaves slack in the second input (i.e., the distance from b to E).

The theoretical literature on these four efficiency measures (see especially Färe et al., 1983; and Russell, 1988) concludes that, for a broad class of reference technologies, they all fail to satisfy all of the desirable properties given above. $DF_i(x,y)$ fails to satisfy (P1) and (P2) (recall the conflict between the Debreu–Farrell and Koopmans notions of efficiency). However, it does satisfy (P3) (homogeneity of degree minus one), and a weaker version of (P4) (i.e., it is weakly, rather than strictly, negative monotonic [20]). $FL_i(x,y)$ satisfies (P1) and (P2), but because of the possibility of output slack, only partially meets the Koopmans definition of efficiency. Furthermore, in general $FL_i(x,y)$ satisfies only weaker versions of (P3) and (P4); it is sub-homogeneous of degree minus one (i.e., the scaling of the input vector by a factor larger [smaller] than unity leads to an efficiency measure smaller [larger] than the inverse scaling of the efficiency measure by the same factor) and is weakly negative monotonic. $Z_i(x,y)$ satisfies (P1), (P2) and (P3), but, again, because of the possibility of output slack, only partially meets the Koopmans definition of efficiency. In general, $Z_i(x,y)$ is nonmonotonic in inputs; i.e., it can either increase or decrease if a single input is increased on some specific technologies. $AF_i(x,y)$ satisfies only (P1); it usually compares inefficient input vectors to the boundary of $L(y)$, not to any of its subsets. In addition, $AF_i(x,y)$ is sub-homogeneous of degree minus one and weakly negative monotonic.

In general, the literature fails to check which properties the various efficiency measures satisfy for the particular reference technology used. For example, if attention is confined to the FDH production technology, the list of satisfied properties changes slightly (Kerstens and Vanden Eeckaut, 1995). Under FDH, $FL_i(x,y)$ does satisfy strict negative monotonicity. But FDH is one of the reference technologies for which $Z_i(x,y)$ is nonmonotonic in inputs. Table 1 summarizes the properties met by the four efficiency measures under FDH.

It should be noted that two additional considerations regarding the choice among technical efficiency measures have appeared in the margin of this literature (see Lovell and Schmidt, 1988). [21] One argument in favor of the Debreu–Farrell efficiency measure is that, as mentioned above, it has a straightforward, factor-price-independent, cost interpretation, which is lacking in the nonradial alternatives. It should be noted, however, that an 'implicit' cost interpretation is possible for the nonradial input efficiency measures. For example, the projection point of $FL_i(x,y)$ results from cost minimization under the assumption that the relative factor prices equal the ratio of the inverse input quantities avail-

[19] This is the same philosophy behind the measures introduced by Lovell and Pastor (1994) and Tone (1993)

[20] Weak monotonicity requires that increasing one input while holding all other inputs and all outputs constant cannot increase the efficiency measure.

[21] Both issues are treated in detail in Ferrier et al. (1994) and Kerstens and Vanden Eeckaut (1995).

Table 1
Properties of the efficiency measures under FDH

	$DF_i(x,y)$	$FL_i(x,y)$	$Z_i(x,y)$	$AF_i(x,y)$
(P1) Indication of Eff L(y)		✓	✓	✓
(P2) Projection to Eff L(y)		✓	✓	
(P3) Homogeneity of degree − 1	✓	SH	✓	SH
(P4) Negative monotonicity	WM	✓		WM
(P5) Units invariance	✓	✓	✓	✓

✓ ≡ property is satisfied;
SH ≡ sub-homogeneity of degree − 1;
WM ≡ weak negative monotonicity.

able to the observation. [22] A second, more theoretical, argument in favor of the Debreu–Farrell efficiency measure is that there exists an equivalence between this efficiency measure and the isoquant of the input correspondence (see Lovell, 1993). However, it can be shown that the nonradial efficiency measures provide similar functional representations of the efficient subset. If the efficient subset is a more important subset than the isoquant for technical efficiency measurement, then this argument would favor the nonradial efficiency measures.

Finally, it is worth mentioning a problem that affects radial efficiency measures in general. Thrall (1989) shows that for the input-based, radial efficiency measure, efficiency scores cannot decrease if additional inputs are added to the model (i.e., if the input dimensionality of the reference technology increases). Hence, while efficient observations remain efficient, inefficient observations may become efficient as the number of input dimension increases. [23] This predictable change of the radial measure leaves room to manipulate the results of any nonparametric efficiency evaluation (Nunamaker, 1985), just as it is possible to do under regression analysis by altering functional form or the set of regressors specified. Kerstens and Vanden Eeckaut (1995) show that the measures $FL_i(x,y)$ and $Z_i(x,y)$ do not change in a monotonic way if additional dimensions are included in the efficiency measurement; in general, $AF_i(x,y)$ does not share this property. This topic requires further attention – our empirical application indicates its importance.

3.4. Radial and nonradial, output and graph measures of technical efficiency

As efficiency measurement relative to the graph of technology is very important under FDH, this section provides the output-oriented and graph counterparts of the radial and the nonradial efficiency measures presented above.

The radial, output-based efficiency measure is formally defined as:

$$DF_o(x,y) = \max\{\mu \mid \mu \geq 1,\ \mu y \in P(x)\}.$$

It measures the maximum proportionate increase in all outputs producible from given inputs. The Färe–Lovell output measure of technical efficiency is defined:

$$FL_o(x,y) = \max\left\{\sum_{i=1}^{n} \mu_i/n \mid (\mu_1 y_1,\ldots,\mu_n y_n) \in P(x), \mu_i \geq 1\right\}.$$

The Zieschang output measure of technical efficiency can be defined as:

$$Z_o(x,y) = FL_o(x, DF_o^+[x,y]\cdot y)\cdot DF_o^+(x,y),$$

[22] Similar 'implicit' cost interpretations for the other nonradial efficiency measures are derived in Kerstens and Vanden Eeckaut (1995).

[23] Taking a different point of view, Charnes and Zlobec (1989) and Charnes and Neralić (1990) address the stability of programming efficiency scores as the reference technology changes due to perturbations of the inputs and outputs in the data set.

where

$$DF_o^+(x,y) = \max\{\mu \mid \mu \geq 1,\ \mu y \in P^+(x) = P(x) + \mathbb{R}_+^n\}.$$

Finally, the asymmetric Färe measure of technical efficiency in the outputs is given by:

$$AF_o(x,y) = \max\{AF_o^j(x,y)\},\quad j = 1,\ldots,n,$$

where

$$AF_o^1(x,y) = \max\{\mu_1 \mid (\mu_1 y_1,\ldots,y_j,\ldots,y_n) \in P(x)\}$$

$$\vdots$$

$$AF_o^n(x,y) = \max\{\mu_n \mid (y_1,\ldots,y_j,\ldots,\mu_n y_n) \in P(x)\}.$$

The interpretations of these four efficiency measures is similar to their input-based counterparts. [24] In the case of output-oriented measures, since slack may exist in the input dimensions, the measures only partially meet the Koopmans definition of efficiency.

There are two graph measures of the Debreu–Farrell type (see Färe et al., 1985, pp. 110–127). The first Debreu–Farrell graph measure of efficiency is:

$$DF_g(x,y) = \min\{\lambda \mid \lambda \geq 0,\ (\lambda x, \lambda^{-1} y) \in GR\}.$$

It gives the maximal equiproportionate reduction of all inputs and increase of all outputs. Note that because inputs and outputs are adjusted simultaneously by the same proportion, the path to the frontier is hyperbolic rather than radial. The generalized Debreu–Farrell graph measure allows the proportionate reduction of all inputs to differ from the proportionate increase of all outputs and averages the two scalars:

$$GDF_g(x,y) = \min\left\{\frac{\lambda + \mu}{2} \mid \lambda \geq 0,\ \mu \geq 0,\ (\lambda x, \mu^{-1} y) \in GR\right\}.$$

The Färe–Lovell graph measure is (see Färe et al., 1985, pp. 153–154): [25]

$$FL_g(x,y) = \min\left\{\left(\sum_{i=1}^{m} \lambda_i + \sum_{j=1}^{n} \mu_j\right)/(m+n) \mid (\lambda_1 x_1,\ldots,\lambda_m x_m, \mu_1^{-1} y_1,\ldots,\mu_n^{-1} y_n) \in GR,\ \lambda_i, \mu_j \in (0,1]\right\}.$$

The Zieschang graph measure is:

$$Z_g(x,y) = FL_g\left(x \cdot DF_g^+[x,y],\ y \cdot DF_g^+[x,y]^{-1}\right) \cdot DF_g^+(x,y),$$

where

$$DF_g^+(x,y) = \min\{\lambda \mid \lambda \geq 0,\ (\lambda x, \lambda^{-1} y) \in GR^+\},$$

where GR^+ is the graph of a technology satisfying strong input and output disposability. Finally, the asymmetric Färe graph measure of technical efficiency is given by:

$$AF_g(x,y) = \min\{AF_g^j(x,y)\},\quad j = 1,\ldots,m+n,$$

where

$$AF_g^1(x,y) = \min\{\lambda_1 \mid \lambda_1 x_1,\ldots,x_m,y_1,\ldots,y_n) \in GR\}$$

$$\vdots$$

$$AF_g^m(x,y) = \min\{\lambda_m \mid (x_1,\ldots,\lambda_m x_m,y_1,\ldots,y_n) \in GR\}$$

$$AF_g^{m+1}(x,y) = \min\{\mu_1 \mid (x_1,\ldots,x_m,\mu_1^{-1} y_1,\ldots,y_n) \in GR\}$$

$$\vdots$$

$$AF_g^{m+n}(x,y) = \min\{\mu_n \mid (x_1,\ldots,x_m,y_1,\ldots,\mu_n^{-1} y_n) \in GR\}$$

[24] In the empirical application below (Section 4), all output-based measures are redefined so as to be situated between zero and one, with unity indicating efficiency. This is common practice in the empirical literature and facilitates the comparison of the various efficiency measures. For example, the Debreu–Farrell measure becomes: $DF_o^*(x,y) = \min\{\mu' \mid 0 < \mu' \leq 1,\ y/\mu' \in P(x)\}$. The definitions of the nonradial efficiency measures can be likewise adapted.

[25] Thanassoulis and Dyson (1992) generalize $FL_g(x,y)$ by allowing each dimension to be adjusted by a different weight. As indicated by Färe et al. (1987), the 'extended' additive model is equivalent to $FL_g(x,y)$. The extended additive model, discussed in Bardhan et al. (1994) and Charnes et al. (1989), among others, leads to a 'measure of efficiency dominance' (MED), guaranteeing units invariance for the additive model of Charnes et al. (1985).

and

$\lambda_j \in (0,1]$ for $j = 1,\dots,m$ and $\mu_k^{-1} \in (0,1]$ for $k = 1,\dots,n$.

Several characteristics of the graph measures are worth noting. First, the graph efficiency measures are slightly more difficult to interpret than their input- and output-oriented counterparts. In physical terms, they indicate the simultaneous input saving and output expansion potential available to inefficient observations. In value terms, they measure a simultaneous reduction in cost and increase in revenue, though no straightforward profit interpretation is possible (see Färe et al., 1985, pp. 107–111, for details).

Second, several special cases are worth noting. If $m = n = 1$, then $\mathrm{GDF_g}(x,y) = \mathrm{DF_g}(x,y)$. While, if $m = n = 1$ and $\lambda = \lambda_1 = \cdots = \lambda_m$ and $\mu = \mu_1 = \cdots = \mu_n$, then $\mathrm{GDF_g}(x,y)(= \mathrm{DF_g}(x,y)) = \mathrm{FL_g}(x,y)$. For $\lambda \neq \mu$, $\mathrm{GDF_g}(x,y) < \mathrm{DF_g}(x,y)$. This is because $\mathrm{GDF_g}(x,y)$ eliminates slack in at least one input and one output dimension, while $\mathrm{DF_g}(x,y)$ can leave slacks in up to $m + n - 1$ dimensions. Furthermore, note that $\mathrm{DF_g}(x,y) \geq \max\{\mathrm{DF_i}(x,y), [\mathrm{DF_o}(x,y)]^{-1}\}$ and that $\mathrm{DF_g}(x,y) = 1$ if either $\mathrm{DF_i}(x,y) = 1$ or $\mathrm{DF_o}(x,y) = 1$ (see Färe et al., 1985). This corroborates the remark at the end of the previous subsection that the radial efficiency measure is sensitive to the number of dimensions evaluated.

Finally, the nonradial graph measures satisfying (P2), $\mathrm{FL_g}(x,y)$ and $\mathrm{Z_g}(x,y)$, project inefficient activities to the graph efficient subset, thereby fully complying with the Koopmans definition of efficiency. Thus, under FDH, $\mathrm{FL_g}(x,y)$ and $\mathrm{Z_g}(x,y)$ relate inefficient observations directly to an observed activity when assessing their performance. From a practical standpoint, this gives $\mathrm{FL_g}(x,y)$ and $\mathrm{Z_g}(x,y)$ an advantage for policy-oriented and managerial purposes, since inefficient observations would have an actual efficient observation available to serve as a role model. In general, the other efficiency measures relate inefficient observations to a projection point on the frontier that is a composite of actual observations. For example, in Fig. 1 the inefficient observation b will be projected by $\mathrm{FL_g}(x,y)$ or $\mathrm{Z_g}(x,y)$ onto one of the dominating observations spanning an orthant (C, D or E). By contrast, for example, the radial input measure would project point b to the unobserved point e, which has the same level of input as observation C but produces less output than C.

3.5. An embarrassment of riches? The choice among efficiency measures

None of the measures considered above possesses clear theoretical superiority over the others. Furthermore, it is unclear whether any of the arguments made in the literature tips the balance in favor of any of the measures for use in empirical work. Given this lack of consensus as to the 'best' measure of efficiency, most practitioners have largely used the traditional input- and output-oriented radial efficiency measures as they offer the advantages of being well-known and easy to compute and are 'real' distances. However, it is precisely because a theoretical solution to the problem of defining an ideal technical efficiency measure has not yet been provided that it is worth asking whether the choice among efficiency measures makes any difference in practice. We think it is worthwhile to give serious consideration to this issue and therefore provide an empirical illustration of these measures on the specific reference technology, FDH.

4. An empirical comparison of efficiency measures on an FDH technology

This section systematically explores whether the choice among the various efficiency measures discussed above makes any difference in practice by studying the technical efficiency of a sample of US banks using an FDH reference technology.

4.1. The sample data

Data on a sample of 575 US depository institutions operating in 1984 are used to calculate the thirteen efficiency measures presented above. The data were collected under the Federal Reserve System's Functional Cost Analysis (FCA) program. The FCA program's aim is to help participating banks to increase their operating efficiency by providing them with average performance figures for similar banks. This feedback assures that participating institutions have a self-interest in reporting data accurately.

Table 2
Descriptive statistics on the sample of US banks

Inputs/outputs	Mean	Standard deviation	Minimum	Maximum
x_1	111.17	130.0	5.10	1165.79
x_2	533145.05	790336.7	2260.13	7608838
x_3	1034901.77	1372993.0	36806.48	1155379.05
y_1	12334.50	15819.4	136	151029
y_2	25470.81	34238.0	226	404045
y_3	2764.97	23965.5	0	570385
y_4	5949.33	10332.9	0	151282
y_5	1476.99	3822.2	0	84515

The appropriate definition and measurement of banking inputs and (especially) outputs is a subject of debate in the literature on bank costs (see Berger and Humphrey, 1992 for a discussion). Most empirical studies now adopt one of two approaches, the 'production' or the 'intermediation' approach (for a thorough discussion of these two approaches, see Colwell and Davis, 1992). The production approach regards banks as producers of deposit and loan accounts using only traditional inputs (e.g., capital and labor). It measures outputs by the numbers of deposit and loan accounts of various types, or by the numbers of transactions carried out on each of these products. Under the intermediation approach, banks collect deposits and purchased funds and intermediate them into various types of loans and other assets. Demand and time deposits are thus viewed as intermediate inputs. In this case the inputs include traditional economic inputs, as well as purchased funds. Therefore, outputs are specified as monetary volumes.

Each of these approaches has its advantages and drawbacks and both have been used in the recent empirical literature on bank performance. For example, Aly et al. (1990), Berger et al. (1987), and Berger and Humphrey (1991) follow the intermediation approach; Ferrier and Lovell (1990) and Fried et al. (1993) opt for the production approach. [26] We adopt the production approach, measuring outputs in terms of numbers of accounts. The outputs specified are the numbers of demand (y_1) and time (y_2) deposit accounts, and the numbers of real estate (y_3), instalment (y_4), and commercial (y_5) loans. The inputs used are the total number of employees (x_1), occupancy and equipment costs (x_2), and expenditures on materials (x_3). [27] Table 2 contains descriptive statistics of these variables. [28]

4.2. Results

FDH uses a vector dominance algorithm to classify observations as either efficient or inefficient (see Tulkens, 1993 for details). An efficient observation is given a score of 1; an inefficient observation's score is calculated relative to the particular observation that dominated it. Of the 575 banks in our data set, 409 are 'undominated'; that is, they are technically efficient relative to the other observations in the data set. All of the efficient observations belong to the efficient subset of the graph correspondence. The remaining 166 observations are 'dominated' by another observation and therefore are classified as technically inefficient. All of the inefficient observations are in the interior of the graph correspondence.

[26] In addition to adopting various approaches to defining and measuring bank inputs and outputs, these studies use a variety of reference technologies. For example, Aly et al. (1990) use variable returns to scale DEA; Ferrier and Lovell (1990) utilize both stochastic parametric frontiers and DEA; Fried et al. (1993) choose the FDH approach. Surveying the empirical literature on bank efficiency, Colwell and Davis (1992) conclude that technical efficiency is more important than any other type of inefficiency.

[27] The measures of capital (x_2) and materials (x_3) are less than ideal; unfortunately, information on the physical quantities of these inputs is not available.

[28] Ferrier and Lovell (1990) analyze the same set of data used in this paper; however, they also include environmental variables in their analysis. Under the nonparametric approach, increasing the number of dimensions reduces the number of technically inefficient observations. To highlight differences across the various efficiency measures as clearly as possible, we choose a specification of the production technology that includes only the inputs and outputs. Therefore, in our analysis environmental variables are neglected, yielding a higher number of technical inefficient observations.

Table 3
Efficiency measures on an FDH reference technology ($N = 575$)

$\mathrm{E}_i(x,y)$	Mean	Standard deviation	Skewness	Kurtosis	Minimum	Maximum
$\mathrm{DF}_i(x,y)$	0.944	0.114	−2.338	8.285	0.391	1.000
$\mathrm{FL}_i(x,y)$	0.879	0.206	−1.372	3.416	0.225	1.000
$\mathrm{Z}_i(x,y)$	0.888	0.192	−1.448	3.755	0.225	1.000
$\mathrm{AF}_i(x,y)$	0.798	0.338	−1.253	2.868	0.007	1.000
$\mathrm{DF}_o(x,y)$	0.949	0.107	−2.410	8.784	0.379	1.000
$\mathrm{FL}_o(x,y)$	0.879	0.205	−1.376	3.480	0.223	1.000
$\mathrm{Z}_o(x,y)$	0.885	0.196	−1.411	3.623	0.225	1.000
$\mathrm{AF}_o(x,y)$	0.797	0.333	−1.161	2.595	0.005	1.000
$\mathrm{DF}_g(x,y)$	0.971	0.065	−2.932	13.337	0.486	1.000
$\mathrm{GDF}_g(x,y)$	0.954	0.086	−2.050	7.216	0.440	1.000
$\mathrm{FL}_g(x,y)$	0.885	0.190	−1.244	2.982	0.306	1.000
$\mathrm{Z}_g(x,y)$	0.893	0.178	−1.273	3.127	0.306	1.000
$\mathrm{AF}_g(x,y)$	0.771	0.369	−1.081	2.328	0.005	1.000

The empirical efficiency scores generated by the various measures are compared in three ways. First, their empirical distributions are examined. Second, the efficiency scores are correlated across the different measures to determine the effect of the choice of measure on individual observations' rankings. Finally, the degree to which the traditional radial efficiency measure approximates the nonradial efficiency measures is examined.

Table 3 reports descriptive statistics for the thirteen input-oriented, output-oriented, and graph efficiency measures calculated for the full sample of data ($N = 575$). In general, the results are as expected: $\mathrm{DF}_i(x,y)$ has the largest mean, followed by $\mathrm{Z}_i(x,y)$, $\mathrm{FL}_i(x,y)$ and $\mathrm{AF}_i(x,y)$. The same ordering of means holds true for the output-based and graph measures. This simply reflects the ranking among efficiency measures mentioned above. $\mathrm{DF}_i(x,y)$ also has the smallest standard deviation and the smallest range, again followed by the $\mathrm{Z}_i(x,y)$, $\mathrm{FL}_i(x,y)$ and $\mathrm{AF}_i(x,y)$. The same pattern holds for the output-oriented and graph measures. All of their distributions are negatively skewed and have positive kurtoses. $\mathrm{DF}_g(x,y)$ is the most pronouncedly skewed and also has the largest kurtosis. The positive kurtoses of the efficiency measures indicates that their distributions have fat tails relative to the normal distribution. The distributions of the same input efficiency measures on the same data set using DEA are shifted strongly downwards; this likely is due to a small number of highly specialized banks found in the data set (see Ferrier et al., 1994). The FDH-based efficiency scores are clearly less vulnerable to such observations.

Note that the radial (Debreu–Farrell) efficiency measures project all 166 inefficient observations onto the isoquant or the weak efficient subset; therefore, the radial and Zieschang measures never coincide. However, the Färe–Lovell and Zieschang input-oriented, output-oriented, and graph efficiency measures were identical in approximately 63% to 68% of the inefficient cases. None of the other efficiency measures coincided for any observations. This in part explains why the Färe–Lovell and Zieschang measures have such similar distributions.

Figs. 3–5 present the density distributions of the

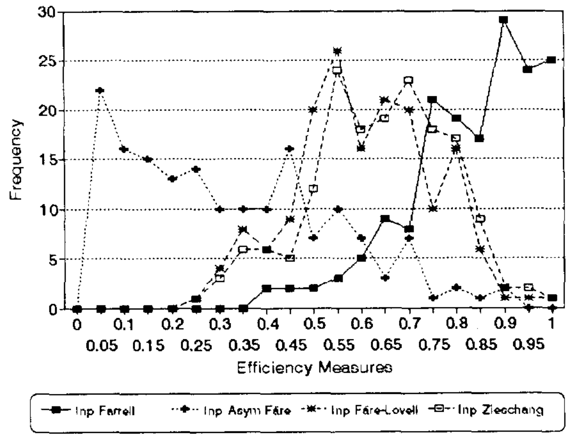

Fig. 3. Densities of input technical efficiency measures on the FDH (inefficient observations only).

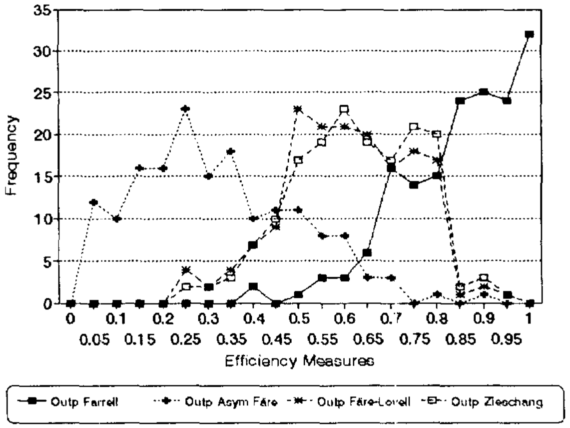

Fig. 4. Densities of output technical efficiency measures on the FDH (inefficient observations only).

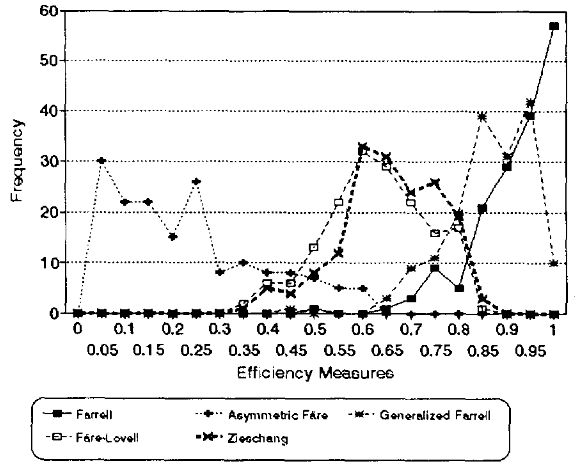

Fig. 5. Densities of graph technical efficiency measures on the FDH (inefficient observations only).

input-oriented, output-oriented, and graph efficiency measures, respectively, based on the inefficient observations only. The distributions appear to differ markedly. These differences are corroborated by two simple nonparametric tests. A Friedman test indicates that the efficiency measures do not follow a common distribution under any of the three orientations. Furthermore, with the exceptions of the pairs $DF_i(x,y) - DF_o(x,y)$, $FL_i(x,y) - FL_o(x,y)$, $Z_i(x,y) - Z_o(x,y)$, $AF_i(x,y) - AF_o(x,y)$, $FL_i(x,y) - FL_g(x,y)$, $Z_i(x,y) - Z_g(x,y)$ and $Z_o(x,y) - FL_g(x,y)$, the Wilcoxon signed-rank test indicates that no pair of efficiency measures shares the same distribution. As these results are reasonable, details on these test statistics are suppressed in the interest of space limitations.

Our results also illustrate the sensitivity of the efficiency measures to the dimensionality of the data, an issue discussed earlier. When comparing the input-, output-oriented, and graph efficiency measures, the total number of dimensions per se does not change. However, since adjustment can occur over both inputs and outputs, the graph measures evaluate efficiency over a larger number of dimensions than do the input- and output-oriented measures. On the one hand, it is clear that the radial measure cannot decrease and the asymmetric Färe measure cannot

Table 4
Correlation matrix across efficiency measures on an FDH reference technology ($N = 166$)

$E_i(x,y)$	$DF_i(x,y)$	$FL_i(x,y)$	$Z_i(x,y)$	$AF_i(x,y)$	$DF_o(x,y)$	$FL_o(x,y)$	$Z_o(x,y)$	$AF_o(x,y)$	$DF_g(x,y)$	$GDF_g(x,y)$	$FL_g(x,y)$	$Z_g(x,y)$	$AF_g(x,g)$
$DF_i(x,y)$	1.000	0.745	0.739	0.443	0.536	0.515	0.484	0.420	0.715	0.849	0.634	0.583	30.99
$FL_i(x,y)$		1.000	0.904	0.831	0.420	0.477	0.441	0.419	0.550	0.623	0.759	0.687	0.679
$Z_i(x,y)$			1.000	0.707	0.399	0.402	0.392	0.324	0.537	0.622	0.666	0.667	0.552
$AF_i(x,y)$				1.000	0.286	0.355	0.327	0.334	0.355	0.385	0.630	0.561	0.775
$DF_o(x,y)$					1.000	0.800	0.796	0.533	0.784	0.846	0.680	0.592	0.375
$FL_o(x,y)$						1.000	0.960	0.833	0.706	0.731	0.897	0.803	0.579
$Z_o(x,y)$							1.000	0.797	0.657	0.708	0.858	0.797	0.565
$AF_o(x,y)$								1.000	0.555	0.551	0.775	0.698	0.695
$DF_g(x,y)$									1.000	0.883	0.683	0.645	0.399
$GDF_g(x,y)$										1.000	0.751	0.704	0.421
$FL_g(x,y)$											1.000	0.912	0.723
$Z_g(x,y)$												1.000	0.646
$AF_g(x,y)$													1.000

increase as variable dimensions are added. The mean of the former increases and its range decreases, while the mean of the latter decreases and its range increases. For the Färe–Lovell and Zieschang efficiency measures, the impact of adding dimensions in the computation of efficiency measures is unclear from the aggregated results.

Additional insight, especially for these nonradial efficiency measures, is achieved by a detailed accounting of the number of decreasing, constant, and increasing efficiency scores among the inefficient observations. Adding dimensions in the calculation of the efficiency measures, for instance moving from the input-oriented to the graph measures, has the following effects in the FDH analysis. The Debreu–Farrell efficiency measure increases in about 70% of the inefficient observations and is constant for the others. The asymmetric Färe efficiency measure decreases for about 50% of the inefficient observations and is constant otherwise. Both the Färe–Lovell and Zieschang efficiency measures increase in about 58% of the cases and decrease in the remaining cases. These results confirm our expectations, with the strong similarity between the Färe–Lovell and the Zieschang efficiency measures due to the fact that both measures account for total input slacks, though each measure follows a different path to the frontier. [29]

Table 4 contains the Pearson product-moment correlations across efficiency measures. Since the correlations based on the full sample are very high due to the high number of efficient observations, the correlations presented in Table 4 are based on only the inefficient observations. In general, the correlations are still relatively high. The highest correlations are those between the two radial graph efficiency measures and between the Färe–Lovell and Zieschang efficiency measures. The latter high correlation is explained in part by the fact that the Färe–Lovell and Zieschang efficiency measures coincide if they relate an observation to a common projection point in the efficient subset. This is not too surprising since both measures partly share the same structure, though it is not obvious a priori that this would imply such similar rankings. Compared with the other measures, the asymmetric Färe efficiency measure has the lowest correlation coefficients. It correlates fairly well with the Färe–Lovell measure, not as well with the Zieschang measure, and the correlation between it and the Debreu–Farrell measure is the weakest in the table.

Finally, it should be noted that similar efficiency measures correlate fairly well across the three orientations, though the correlation between the input- and output-orientations is rather low; e.g., the correlation between $AF_i(x,y)$ and $AF_o(x,y)$ is only 0.334. In general, dissimilar measures correlate much better within an orientation than they do across orientations. The correlations across orientations are lowest when comparing the ranking of the input- and output-orientations; this is likely due to the presence of nonconstant returns to scale. Overall, this indicates that, at least under the FDH reference technology, both choice of measure and choice of orientation impact upon efficiency rankings. This opens another avenue for the manipulation of efficiency rankings in empirical analyses. An unscrupulous practitioner could select the measure and orientation that sheds the most favorable light upon his/her preferred observations. Thus, if performance ranking is a primary objective, or if there is uncertainty about organizational objectives, it seems advisable to perform a sensitivity analysis with regard to the choice of measure and the orientation of efficiency measurement.

The distinction between the isoquant and the efficient subset is important in theory. Furthermore, differences among the various efficiency measures exist both in theory and in practice. However, it may the case that in practice the Debreu–Farrell measures serve as 'good' approximations to the nonradial efficiency measures. If Debreu–Farrell measures scale down the inefficient observations 'close' to the efficient subset, then the choice to use them over one of the alternatives may not be of much consequence. It is therefore worthwhile to assess the Debreu–Farrell measures' powers of approximation relative to the efficient subset for the banks in our sample. For this purpose we use the terminology of Fried et al. (1993), Lovell (1993, 1992), and Lovell and Vanden Eeckaut (1994) who suggest reporting any remaining

[29] Recall the earlier observation that the pairs $FL_i(x,y)$ and $Z_i(x,y)$ and $FL_g(x,y)$ and $Z_g(x,y)$ follow a common distribution.

Table 5
Slacks and radial efficiency in the inputs ($N = 166$)

Dimension	Mean	Standard deviation	Maximum	Minimum
Total Slack (%)				
Input 1	54.31	24.93	0.78	99.32
Input 2	21.70	14.11	0.09	60.91
Input 3	40.07	21.00	0.02	94.68
Output 1	23.97	27.47	0.04	190.10
Output 2	85.57	105.78	0.53	1023.00
Output 3	174.82	364.82	0	3018.00
Output 4	105.92	183.70	0.78	1418.00
Output 5	153.66	461.68	0	5348.00
Slack eliminated by the radial efficiency measure (%)				
All inputs	19.33	13.71	0.03	60.90
Slack not eliminated by the radial efficiency measure (%)				
Input 1	34.99	24.54	0	94.51
Input 2	2.38	5.75	0	27.41
Input 3	20.74	17.44	0	75.23

'slacks' when using radial efficiency measures on the FDH technology. [30]

'Total slack' per dimension is defined as the difference in input and output usage between the evaluated observation and its most dominating observation (i.e., the dominating observation in the efficient subset against which its efficiency is measured). Slacks therefore refer to the excessive utilization of inputs and/or the underprovision of outputs. This 'total slack' can be decomposed into radial and nonradial components. The 'radial slack' denotes the difference between the evaluated observation and the projection point of the radial efficiency measure on the isoquant. The 'nonradial slack' equals the 'total slack' minus the 'radial slack'. Fig. 2 illustrates these notions of slack. Note that because slack is measured in the original units of the input and output variables, meaningful comparisons are made possible by expressing slack as a percentage of the observed values.

Table 5 illustrates the problem of slacks for the radial, input-oriented measure of technical efficiency. [31] Observe that on FDH the radial, input-based measure may leave 'nonradial slacks' in up to $m - 1$ input and in all n output dimensions. As all 166 inefficient observations are scaled down to the isoquant (or weak efficient subset) of the input correspondence, the 'total slacks' are rather important, averaging 40% of the initial input dimensions and 110% of the initial output dimensions. In general the range is wide, especially in the output dimensions. The radial measure partially eliminates the 'total slack'. 'Radial slack' averages only 19% of the 'total slack' in each input dimension, with a maximum value of about 61%. The 'nonradial', or remaining, slack is more important in two of the three input

[30] As noted by a referee, this may be cumbersome and is not entirely satisfying because it fails to succinctly summarize the true level of efficiency realized by an observation. The non-Archimedean element in DEA or the measure $Z_i(x,y)$ both remove slack in their calculations, thus eliminating the need to treat slack as a separate issue.

[31] As noted by a referee, if computation of an efficiency measure yields alternate optima, then it is important to compute the associated slacks so as to determine the correct projection point (see Bardhan et al., 1994). In our sample, only two observations had alternative optima for the radial output and/or graph efficiency measures.

dimensions. Only for the second input dimension (i.e., capital) does the radial efficiency measure manage, on average, to eliminate most of the 'total slack' in production.

This result is in line with what one would expect, and it is consistent with the findings of pervasive remaining slacks reported in Fried et al. (1993) and Lovell (1992). It appears that the radial efficiency measure poorly bridges the gap between inefficient observations and the efficient subset. It is therefore serves as a poor approximation of the nonradial efficiency measures. Furthermore, it is clear that in the case of the FDH reference technology, restricting attention to the input orientation of measurement may leave a lot of unmeasured slack in the output dimensions. This result clearly illustrates the usefulness of graph efficiency measurement on FDH.

5. Summary and conclusions

The purpose of this paper was twofold. First, the choice among measures as well as orientation for assessing technical efficiency was analyzed from a theoretical viewpoint. A review of the axiomatic literature provided a list of desirable properties that an 'ideal' measure of technical efficiency would possess. It also suggested three nonradial alternatives to the standard radial efficiency measure of Debreu–Farrell. Both the Debreu–Farrell measure and its rivals were presented for input, output, and graph orientations. Unfortunately, none of these measures satisfies all of the desirable properties. The second purpose of the paper was to compare the performance of these various measures of technical efficiency on a common set of data using the FDH approach. FDH is an attractive deterministic-nonparametric reference technology for the evaluation of productive efficiency. Furthermore, FDH accentuates the differences between the radial and nonradial efficiency measures, therefore providing a good test case for examining the practical importance of the choice among alternative efficiency measures and orientations. The empirical example reveals wide differences in the distributions of efficiency scores and in the resulting correlations across alternative measures and orientations. It also demonstrates that the radial and the nonradial measures are not close empirical substitutes, as the radial measures tend to project inefficient observations points that are far removed from the efficient subset of technology.

Two main conclusions emerge from the analysis. First, because the efficient subset is relatively small for the FDH reference technology, the choice among various efficiency measures is of crucial importance in measuring technical efficiency. In particular, our empirical illustration indicates that the radial efficiency measure does a poor job of closing the distance between inefficient observations and the efficient subset. For the FDH reference technology, the graph oriented measures of efficiency appear to be helpful in complying with Koopmans definition. Second, both a priori theoretical arguments and the empirical evidence resulting from analyzing a sample of US banks suggest that the Färe–Lovell and Zieschang efficiency measures may provide valuable alternatives to the standard radial measures of Debreu–Farrell. In addition, the recently developed measures of efficiency that account for both input and output slacks (e.g., those of Pastor, 1995 and Tone, 1993) offer promising alternatives that should be considered in future work.

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